2023 MOCK 1 ELECTIVE MATHEMATICS 2 SOLUTION

QUESTION NUMBER	DETAILS	MARKS
1. (a)	$m\Delta n = m + n + 10$ Let e be the identity element $m\Delta e = m$ $m + e + 10 = m$ $e = -10$	M_1 for finding the identity element A_1 for $e = -10$
(b)	Let m^{-1} be the inverse of m $m\Delta m^{-1} = e$ $m + m^{-1} + 10 = e$ $m^{-1} = -10 - 10 - m$ $m^{-1} = -20 - m$ ∴the inverse of $2 = -20 - 2$ $= -22$ Inverse of $-5 = -20 - (-5)$ $= -15$	M_1 for substitution A_1 for $-20 - m$, inverse of m A_1 for -22 A_1 for -15 **[6 marks]**
2.	$log(x-1) + 2\,log\,y = 2\,log\,3$ $log(x-1) + log\,y^2 = log\,9$ $y^2(x-1) = 9$ ______ 1 *Again* $log\,x + log\,y = log\,6$ $log\,x\,y = log\,6$ Taking antilog of both sides, $xy = 6$ ______ 2 from 2; $y = \frac{6}{x}$ ______ 3 putting 3 into 1; $(\frac{6}{x})^2\,(x-1) = 9$ $9x^2 - 36x + 36 = 0$ or $x^2 - 4x + 4 = 0$ $(x-2)(x-2) = 0$ $x = 2$ When $x = 2, y = \frac{6}{2}$ $y = 3$	B_1 for removing log in either equation 1 or M_1 for solving the equations A_1 either of the equations $x^2 - 4x + 4 = 0$ M_1 for solving for x A_1 for $x = 2$ A_1 for $y = 3$ **[6 marks]**
3.	From $4x^2 + 7x + 3 = 0$ From $ax^2 + bx + c = 0$, $a = 4,\ \ b = 7,\ \ c = 3$ $\alpha + \beta = -\frac{b}{a} = -\frac{7}{4}$	B_1 for sum of roots

	$\alpha\beta = \frac{c}{a} = \frac{3}{4}$ $\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2+\beta^2}{\alpha^2\beta^2} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$ $\frac{(-\frac{7}{4})^2 - 2(\frac{3}{4})}{(\frac{3}{4})^2}$ $\frac{\frac{49}{16} - \frac{6}{4}}{\frac{9}{16}}$ $= \frac{25}{9}$	B_1 for product of roots $M_1 A_1$ for correct substitution and all entries correct M_1 for simplification $A_1 = \frac{25}{9}$ **[6 marks]**
4.	The nth term is given by $U_n = S_n - S_{n-1}$ $U_n = S_n - S_{n-1} = \frac{5}{2}n^2 + \frac{5}{2}n - (\frac{5}{2}n^2 - \frac{5}{2}n)$ $U_n = \frac{5}{2}n^2 + \frac{5}{2}n - \frac{5}{2}n^2 + \frac{5}{2}n$ $U_n = \frac{10n}{2} \to U_n = 5n$ The first four terms are. $U_1 = 5(1)\ U_1 = 1,\quad U_2 = 5(2) = 10$ $U_3 = 5(3) = 15,\quad U_4 = 5(4) = 20$	M_1 for the expression M_1 for the simplifying A_1 for finding nth term U_n M_1 for substitution A_2 for all 4 terms 5, 10, 15, 20 Consider other methods **[6 marks]**
5. (a) (b)	P (both of same group) $= \frac{7}{12} \times \frac{6}{11} + \frac{5}{12} \times \frac{4}{11}$ $= \frac{42}{132} + \frac{20}{132}$ $= \frac{31}{66}$ Or $\frac{7c_2}{12c_2} + \frac{5c_2}{12c_2} = \frac{31}{66}$ P (both of different group) $= \frac{7}{12} \times \frac{5}{11} + \frac{5}{12} \times \frac{7}{11}$ $= \frac{35}{132} + \frac{35}{132}$ $= \frac{35}{66}$	B_1 for substitution M_1 for simplification A_1 for $= \frac{31}{66}$ B_1 for substitution M_1 for simplification A_1 for $= \frac{35}{66}$

Or $\frac{7c_1 \times 5c_1}{12c_2} = \frac{35}{66}$

Consider other alternatives

[6 marks]

6.

P	Q	R_P	R_Q	$d = R_P - R_Q$	d^2
0	0	10	10	0	0
7	107	3	1	2	4
3	25	7	8	-1	1
5	101	5	3	2	4
6	63	4	6	-2	4
1	81	9	5	4	16
8	42	2	7	-5	25
9	94	1	4	-3	9
4	12	6	9	-3	9
2	105	8	2	6	36

B_1 for $R_P(-\frac{1}{2}ee)$

B_1 for $R_Q(-\frac{1}{2}ee)$

B_1 for $d^2(-\frac{1}{2}ee)$

$\Sigma d^2 = 108$

B_1 for $\Sigma d^2 = 108$

$n = 10$

$r = 1 - \frac{6 \times 108}{10(100-1)}$

M_1 for subtracting and simplifying.

$= 1 - \frac{648}{990}$

$r = 0.3455$

$A_1$for 0.3455 only

[6 marks]

7. (a)

Let $A(4,2), B(5,3)$ and $C(2,5)$ be vertices of triangle ABC
Finding the length of AB, BC and AC respectively

M_1 for finding the length of AB, AC and BC

$$|AB| = \sqrt{(3-2)^2 + (5-4)^2}$$
$$= \sqrt{2} \text{ unit}$$

A_1 for $\sqrt{2}$

$$|AC| = \sqrt{(5-2)^2 + (2-4)^2}$$
$$= \sqrt{13} \text{ unit}$$

A_1 for $\sqrt{13}$

$$|BC| = \sqrt{(5-3)^2 + (2-5)^2}$$
$$= \sqrt{13} \text{ unit}$$

A_1 for $\sqrt{13}$
Hence $|AC| = |BC| = \sqrt{13}$,
∴Triangle ABC is isosceles

(b)

For orthogonal vectors $a.b = 0$

$$(k \times 2 \times) + (2 \times (-4)) = 0$$
$$k = 4$$

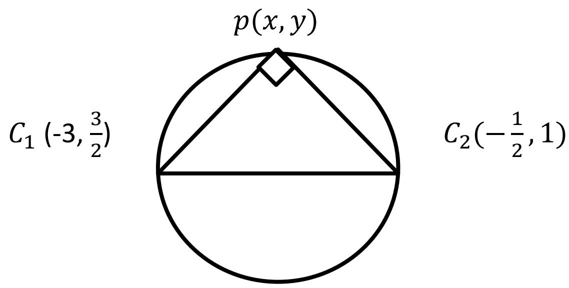
M_1 for applying dot product
A_1 for $k = 4$
Consider vector approach

[6 marks]

8.	The resultant force is $$F = 15\times\frac{1}{5}\begin{pmatrix}-3\\4\end{pmatrix} + 9\times\frac{1}{13}\begin{pmatrix}13\\0\end{pmatrix} + 34\times\frac{1}{17}\begin{pmatrix}8\\-15\end{pmatrix}$$ $$= \begin{pmatrix}-9\\12\end{pmatrix} + \begin{pmatrix}9\\0\end{pmatrix} + \begin{pmatrix}16\\-30\end{pmatrix}$$ $$= \begin{pmatrix}16\\-18\end{pmatrix} \text{ or } 16j - 18j$$ Magnitude of the resultant force $$\lvert F\rvert = \sqrt{256+324}$$ $$= \sqrt{580}$$ $$= 24.1N$$ F is in the 4th quadrant. Let θ= the angle F marks with the $x-axis$ $$\tan\theta = \frac{18}{16}$$ $$\theta = 48.4^0$$ ∴the direction of the resultant force is equal to $$90 + 48.4^0 = 138.4^0$$	M_1 for resolving the forces A_1 for $\begin{pmatrix}16\\-18\end{pmatrix}$ or $16j - 18j$ M_1 for finding $\lvert F\rvert$ A_1 for $24.1N$ M_1 for finding θ A_1 for 138^0 Accept 138^0 **[6 marks]**
9. (a)	$\frac{3x^2+7x+28}{x(x^2+x+7)} = \frac{A}{x} + \frac{Bx+C}{x^2+x+7}$ $$3x^2 + 7x + 28 = A(x^2+x+7) + x(Bx+C)$$ $$3x^2 + 7x + 28 = Ax^2 + Ax + 7A + Bx^2 + Cx$$ 7A=28 ∴A=4 A+B=3 4+B=3 ∴B=-1 A+C=7 ∴C=3 ∴$\frac{3x^2+7x+28}{x(x^2+x+7)} = \frac{4}{x} + \frac{3-x}{(x^2+x+7)}$	B_1 for $\frac{A}{x} + \frac{Bx+c}{(x^2+x+7)}$ M_1 for equating numerator M_1 for finding constants A_1 for $A = 4$ A_1 for $B = -1$ A_1 for $C = 3$ A_1 for $\frac{3x^2+7x+28}{x\,(x^2+x+7)} = \frac{A}{x} + \frac{3-x}{(x^2+x+7}$
(b)	$$7x^2 - 8y^2 + 2xy - 3x = 101$$ $$14x - 16y\frac{dy}{dx} + 2y + 2x\frac{dy}{dx} - 3 = 0$$	M_1 for diffrentiating(At least one term cor

	$(2x-16y)\frac{dy}{dx}=3-2y-14x$	A_1 for correct differentiation
	$\frac{dy}{dx}=\frac{3-2y-14x}{2x-16y}$	$M_1\ for\ \frac{dy}{dx}$
	$\frac{dy}{dx}(-2,5)=\frac{3-2(5)-14(-2)}{2(-2)-16(5)}$	A_1 for $\frac{dy}{dx}=\frac{3-2y-14x}{2x-16y}$
	$=\frac{3-10+28}{-4-80}$	M_1 for substituting
	$=-\frac{1}{4}$	A_1 for $-\frac{1}{4}$
		[13 marks]
10. (a) (i)	$A=\begin{pmatrix}-2 & -1\\ -5 & 3\end{pmatrix}\quad B=\begin{pmatrix}5 & 1\\ 2 & 3\end{pmatrix}$	B_1 for $A=\begin{pmatrix}-2 & -1\\ -5 & 3\end{pmatrix}$
	$A^2=\begin{pmatrix}-2 & -1\\ -5 & 3\end{pmatrix}\begin{pmatrix}-2 & -1\\ -5 & 3\end{pmatrix}=\begin{pmatrix}4+5 & 2-3\\ 10-15 & 5+9\end{pmatrix}$	M_1 for finding either A^2 or B^2
	$=\begin{pmatrix}9 & -1\\ -5 & 14\end{pmatrix}$	A_1 for either $A^2=\begin{pmatrix}9 & 1\\ -5 & 14\end{pmatrix}$ or
	$B^2=\begin{pmatrix}5 & 1\\ 2 & 3\end{pmatrix}\begin{pmatrix}5 & 1\\ 2 & 3\end{pmatrix}=\begin{pmatrix}25+2 & 5+3\\ 10+6 & 2+9\end{pmatrix}$	
	$=\begin{pmatrix}27 & 8\\ 16 & 11\end{pmatrix}$	$B^2=\begin{pmatrix}27 & 8\\ 16 & 11\end{pmatrix}$
(ii) α.	$B^2-A^2=\begin{pmatrix}27 & 8\\ 16 & 11\end{pmatrix}-\begin{pmatrix}9 & -1\\ -5 & 14\end{pmatrix}$	M_1 for subtracting
	$=\begin{pmatrix}18 & 9\\ 21 & -3\end{pmatrix}$	A_1 for $\begin{pmatrix}18 & 9\\ 21 & -3\end{pmatrix}$
β.	$H=\begin{pmatrix}27 & 8\\ 16 & 11\end{pmatrix}\begin{pmatrix}-2 & -1\\ -5 & 14\end{pmatrix}$	M_1 for multiplying
	$=\begin{pmatrix}27(-2)+8(-5) & 27(-1)+8(3)\\ 16(-2+11(-5)) & 16(-1)+1(3)\end{pmatrix}$	M_1 for simplification
(b)	$=\begin{pmatrix}-94 & -3\\ -84 & 17\end{pmatrix}$	$A_1\ for\ H=\begin{pmatrix}-94 & -3\\ -84 & 17\end{pmatrix}$

	$\dfrac{\frac{3}{2}\log 27 - 3\log 5\sqrt{5}}{\log 0.6} = \dfrac{\frac{3}{2}\log 3^3 - 3\log 5^{3/2}}{\log\frac{3}{5}}$	M_1 A_1 for expressing the log
	$= \dfrac{\frac{3}{2}\log 3^3 - \log 5^{9/2}}{\log 3 - \log 5}$	M_1 for correct expression of the log (at least) one term correct
	$= \dfrac{\frac{9}{2}\log 3 - \frac{9}{2}\log 5}{\log 3 - \log 5}$	M_1 for simplification
	$= \dfrac{9}{2}\,\dfrac{(\log 3 - \log 5)}{\log 3 - \log 5} \quad = \dfrac{9}{2}$	A_1 for $\dfrac{9}{2}$ or 4.5
		[13 marks]
11. (a)	Comparing $x^2 + y^2 + 6x + 3y + 4 = o$ to the standard equation of the circle, $g = 3, f = \frac{3}{2}$, hence centre of the circle $C_1(-g, -f)$ $C_1\ (-3, -\frac{3}{2})$	$B_1$for finding centre $C_1(-3, -\frac{3}{2})$
	From $x^2 + y^2 + x - 2y - 3 = 0$, $g = \frac{1}{2}$ $f = -1$ Hence centre of circle $c_2(-\frac{1}{2}, 1)$	$B_1$for finding centre $C_2(-\frac{1}{2}, 1)$
(i)	Gradient of the gradient joining the centres	M_1 for finding the gradient
	$= \dfrac{1 - (-\frac{3}{2})}{-\frac{1}{2} - (-3)}$	
	$= 1$	A_1 for the gradient
	Hence equation of the line using $(-\frac{1}{2}, 1)$ $y - 1 = 1(x + \frac{1}{2})$	
	$y - 1 = x + \dfrac{1}{2}$	
	$2y - 2x - 3 = 0$	A_1 for obtaining the equation of the line
(ii)	Let $C_1\left(-3, -\frac{3}{2}\right), C_2\left(-\frac{1}{2}, 1\right)$ and $P(x, y)$	

	Be any point on the circle  If $C_1\ C_2$ is the diameter, then C_1P is perpendicular to PC_2 Gradient of C_1P $\times$ Gradient of $PC_2 = -1$	
	$$\frac{y+\frac{3}{2}}{x+3} \times \frac{y-1}{x+\frac{1}{2}} = -1$$	M_1 for the main expression
	$$\frac{y^2 - y + \frac{3}{2}y - \frac{3}{2}}{y^2 + \frac{1}{2}x + 3x + \frac{3}{2}} = -1$$	
	$$y^2 + \frac{1}{2}y - \frac{3}{2} = -1(x^2 + \frac{7}{2}x + \frac{3}{2})$$	M_1 for simplifying
	$$y^2 + \frac{1}{2}y - \frac{3}{2} = -x^2 - \frac{7}{2}x - \frac{3}{2}$$ Multiplying through by 2	
	$$2x^2 + 2y^2 + 7x + y = 0$$	A_1 for obtaining the equation
(b)	From $3x^2 - 5x - 1 = 0$ $a = 3, \quad b = -5, \quad c = -1$	
	$\alpha + \beta = \frac{5}{3} \qquad \alpha\beta = -\frac{1}{3}$	B_1 for either sum or product of roots
	Sam the roots: $\left(\alpha - \frac{1}{\beta}\right) + \left(\beta - \frac{1}{\alpha}\right) = (\alpha + \beta) - (\frac{\alpha+\beta}{\alpha\beta})$	

$$= \frac{5}{3} - \left(\frac{5/3}{-1/3}\right)$$

M_1 for substitution

$$= \frac{20}{3}$$

Product of roots: $\left(\alpha - \frac{1}{\beta}\right)\left(\beta - \frac{1}{\alpha}\right)$

$$= \alpha\beta + \frac{1}{\alpha\beta} - 2$$

A_1 for either $\frac{20}{3}$ Or $\frac{16}{3}$

$$= \frac{16}{3}$$

M_1 for forming the equation

Equation is $x^2 - \frac{20}{3}x - \frac{16}{3} = 0$

A_1 for obtaining the equation

$$3x^2 - 20x - 16 = 0$$

[13 Marks]

12.

Marks	Freq(f)	Midpt(x)	Deviation $d = x - A$	fd	fd^2
21-30	2	25.5	-40	-80	3200
31-40	7	35.5	-30	-210	6300
41-50	6	45.5	-20	-120	2400
51-60	7	55.5	-10	-70	700
61-70	8	65.5	0	0	0
71-80	5	75.5	10	50	500
81-90	3	85.5	20	60	1200
91-100	2	95.5	30	60	1800
	Σf=40			Σfd= -310	Σfd^2 =16100

B_1 for Σf=40

B_2 for $\Sigma fd(1/2\, ee)$

B_2 for midpoints $(1/2\, ee)$

B_2 for $\Sigma fd^2(1/2\, ee)$

Using the assume mean of 65.5

(a)

Mean $\overline{x} = A + \frac{\Sigma fd}{\Sigma f}$

$$= 65.5 + \left(\frac{-130}{40}\right)$$

M_1 for substitution

$$= 65.5 - 7.75$$

M_1 for simplification

$$= 57.75$$

A_1 for finding the mean $= 57.75$

(b)

Standard deviation

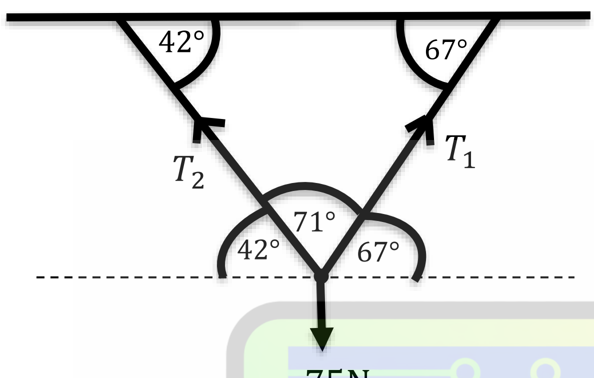
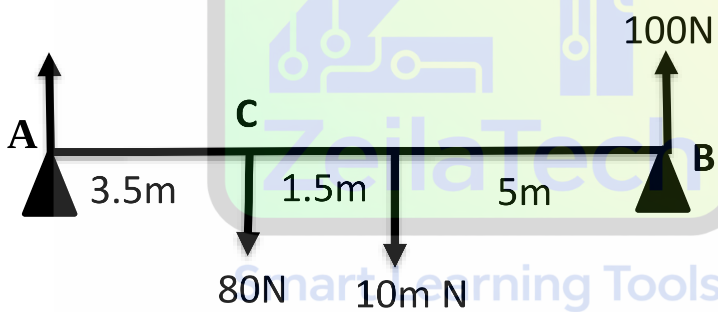
$$SD = \sqrt{\frac{\Sigma fd^2}{\Sigma f} - \left(\frac{\Sigma fd}{\Sigma f}\right)^2}$$

M_1 for finding SD

	$= \sqrt{\frac{16100}{40} - (\frac{-310}{40})^2}$ $= \sqrt{342.4375}$ $= 18.505$	M_1 for simplying A_1 for $SD = 18.505$ **[13 marks]**
13. (a) (i) (ii)	P (they are of the same colour) $= \left(\frac{7}{11} \times \frac{6}{10} \times \frac{5}{9}\right) + \left(\frac{4}{11} \times \frac{3}{10} \times \frac{2}{9}\right)$ $= \frac{7}{33} + \frac{4}{165}$ $= \frac{13}{55}$ $= 0.24$ Alternatively P (same colour)$= \frac{7_{C3}}{11_{C3}} + \frac{11_{C3}}{11_{C3}}$ $= 0.24$ P (two is blue and one red) $= \left(\frac{7}{11} \times \frac{6}{10} \times \frac{5}{9}\right) + \left(\frac{7}{11} \times \frac{4}{10} \times \frac{6}{9}\right) + \left(\frac{4}{11} \times \frac{7}{10} \times \frac{6}{9}\right)$ $= \frac{168}{990} + \frac{168}{990} + \frac{168}{990}$ $= \frac{504}{990}$ $= \frac{28}{55}$ $= 0.51$ Alternatively, P (two blue and one red) $= \frac{7_{C2} \times 11_{C2}}{11_{C3}}$	 M_1 for substitution M_1 for simplifying A_1 for $= 0.24$ insist on 2dp M_1 for correct substitution (all entries) M_1 for simplifying A_1 for $= 0.51$ (Accept 2dp only)

	$= \dfrac{21 \times 4}{165}$ $= 0.51$	
(b)	P (attending AOGA meeting) $p = \frac{3}{8}$ P (not attending AOGA meeting)$q = 1 - \frac{3}{8} = \frac{5}{8}$ Let x = the number of students attending AOGA meeting and the probability function is defined by $P(x) = 5_{Cx}\,(\dfrac{3}{8})^{x}\,(\dfrac{5}{8})^{5-x}$	B_1 for any of the probabilities
(i)	P (exactly two) $= P(x = 2)$ $= 5_{C2}\,(\dfrac{3}{8})^{2}\,(\dfrac{5}{8})^{3}$ $= 0.34$	M_1 for substitution A_1 for answer 0.34 insist on 2dp
(ii)	P (less than half) $= p(x \leq 2)$ $p = (x = 0) + p(x = 1) + p(x = 2)$ $= 5_{C0}\,(\dfrac{3}{8})^{0}\,(\dfrac{5}{8})^{5} + 5_{C1}\,(\dfrac{3}{8})^{1}\,(\dfrac{5}{8})^{4} + 5_{C2}\,(\dfrac{3}{8})^{2}\,(\dfrac{5}{8})^{3}$ $= \dfrac{3125}{32758} + \dfrac{9375}{32768} + \dfrac{5625}{32768}$ $= 0.72$ P(at least three) $= 1 - P(x \leq 2)$ $1 - 0.72$ $= 0.28$	M_1 for substitution A_1 for answer 0.72 M_1 for substitution A_1 for answer 0.28 **[13 marks]**
14. (a)	$\overrightarrow{AB} = \binom{-1}{3} - \binom{3}{-2}$ $= \binom{-4}{5}$ $\overrightarrow{AB} = \sqrt{(4)^2 + 5^2}$	B_1 for $\binom{-4}{5}$

	$=\sqrt{16+25}$	
	$=\sqrt{41}$	B_1 for $\sqrt{41}$
	Unit vector of $\overrightarrow{AB} = \frac{1}{\sqrt{41}}(-4i+5j)$	M_1 for finding unit vector
	$\overrightarrow{AB} = \frac{\sqrt{41}}{41}(-4i+5j)$	A_1 for $\frac{\sqrt{41}}{41}(-4i+5j)$
(b)	If angle $AOC = 90^0$, $\overrightarrow{OA}.\overrightarrow{OC} = 0$	
	$\binom{3}{2}.\binom{4}{n} = 0$ $12-2n=0$ $n=6$	M_1 for finding n A_1 for $n=6$
(c)	$\overrightarrow{AD} = \binom{p}{-1} - \binom{3}{-2}$	
	$= \binom{p-3}{1}$	M_1 for subtracting vectors A_1 for $\binom{p-3}{1}$
	$\overrightarrow{AD} = \sqrt{37}$units, $\sqrt{(p-3)^2+1^2} = \sqrt{37}$	M_1 for subtracting
	$(p-3)^2+1=37$	M_1 for simplifying
	$(p-3)^2 = 36$ $p-3=\pm 6$	A_1 for quadratic M_1 for solving
	$p = 3-6$ or $p=3+6$	
	$p=-3$ or 9	A_1 for $p=-3$ or $9\left(-\frac{1}{2}ee\right)$ **[13 marks]**
15. (a)	let T_1 *and* T_2 be the tensions in the rope from Lami's theorem	
	$\frac{T_1}{\sin(180-132)} = \frac{T_2}{\sin(180-157)} = \frac{75}{\sin(180-71)}$	M_1 for subtracting
	$T_1 = \frac{75\sin 48^0}{\sin 109^0}$	M_1 for finding T_1
	$=58.95N$	A_1 for $58.95N$ (insist in 2 dp)

	Also $T_2 = \frac{75\sin 23^0}{\sin 109^0}$	M_1 for finding T_2
	$T_2 = 30.99N$	A_1 for $30.99N$
		$B_1\left(-\frac{1}{2}ee\right)$ for the diagram
(b)		$B_1\left(-\frac{1}{2}ee\right)$ for the diagram
(i)	Taking moments about A $(80 \times 3.5) + (10m \times 5) = 100 \times 10$ $280 + 50m = 100$ $50m = 720$ $m = \frac{720}{50}$ $m = 14.4kg$	M_1 for correct substitution M_1 for solving for the mass m A_1 for $14.4kg$

(ii)	Total upward forces = Total downward forces $R_A + 100 = 80 + (10 \times 14.4)$ $R_A + 100 = 80 + 144$ $R_A + 100 = 224$ $R_A = 224 - 100$ $R_A = 124N$	M_1 for substitution A_1 for $124N$ (-1 $o.u$ or $w.u$ once only) **[13 marks]**